# Ecuaciones Diferenciales Parciales y Cálculo de Series de Fourier

## 1 Definición de los operadores de coeficientes de Fourier

(%i151) a(f,n,L):=integrate(f*cos(2*n*%pi*x),x,0,L)*2/L;

$$\mathrm{a}\left(f, n, L\right) := \frac{\int_{0}^{L} f\cos\left(2n\pi x\right)dx\,2}{L} \qquad (\%\ \mathrm{o}151)$$

(%i152) b(f,n,L):=integrate(f*sin(2*n*%pi*x),x,0,L)*2/L;

$$\mathrm{b}\left(f, n, L\right) := \frac{\int_{0}^{L} f\sin\left(2n\pi x\right)dx\,2}{L} \qquad (\%\ \mathrm{o}152)$$

## 2 Definición ("elegante") del operador de representación de Fourier

(%i153) F(f,N,L):=radcan(trigsimp(a(f,0,L)))/2+ sum(radcan(trigsimp(a(f,n,L)))*cos(2*n*%pi*x),n,1,N)+ sum(radcan(trigsimp(b(f,n,L)))*sin(2*n*%pi*x),n,1,N);

$$\mathrm{F}\left(f, N, L\right) := \frac{\mathrm{radcan}\left(\mathrm{trigsimp}\left(\mathrm{a}\left(f, 0, L\right)\right)\right)}{2} + \sum_{n=1}^{N} \mathrm{radcan}\left(\mathrm{trigsimp}\left(\mathrm{a}\left(f, n, L\right)\right)\right)\cos\left(2n\pi x\right) + \sum_{n=1}^{N} \mathrm{radcan}\left(\mathrm{tr}\right.$$

(% o153)

## 3 Definición (semi-numérica) de operador de representación de Fourier

(%i154) a_n(f,n,L):=quad_qag(f*cos(2*n*%pi*x),x,0,L,1)[1]*2/L;

$$\mathrm{a\_n}(f,n,L):=\frac{[\mathrm{quad\_qag}]_1 2}{L} \qquad (\%\ o154)$$

(%i155) b_n(f,n,L):=quad_qag(f*sin(2*n*%pi*x),x,0,L,1)[1]*2/L;

$$\mathrm{b\_n}(f,n,L):=\frac{[\mathrm{quad\_qag}]_1 2}{L} \qquad (\%\ o155)$$

(%i156) Fn(f,N,L):=a_n(f,0,L)/2+
sum(a_n(f,n,L)*cos(2*n*%pi*x),n,1,N)+
sum(b_n(f,n,L)*sin(2*n*%pi*x),n,1,N);

$$\mathrm{Fn}(f,N,L):=\frac{\mathrm{a\_n}(f,0,L)}{2}+\sum_{n=1}^{N}\mathrm{a\_n}(f,n,L)\cos(2n\pi x)+\sum_{n=1}^{N}\mathrm{b\_n}(f,n,L)\sin(2n\pi x) \qquad (\%\ o156)$$

## 4 Ejemplos de cálculo de representación de Fourier

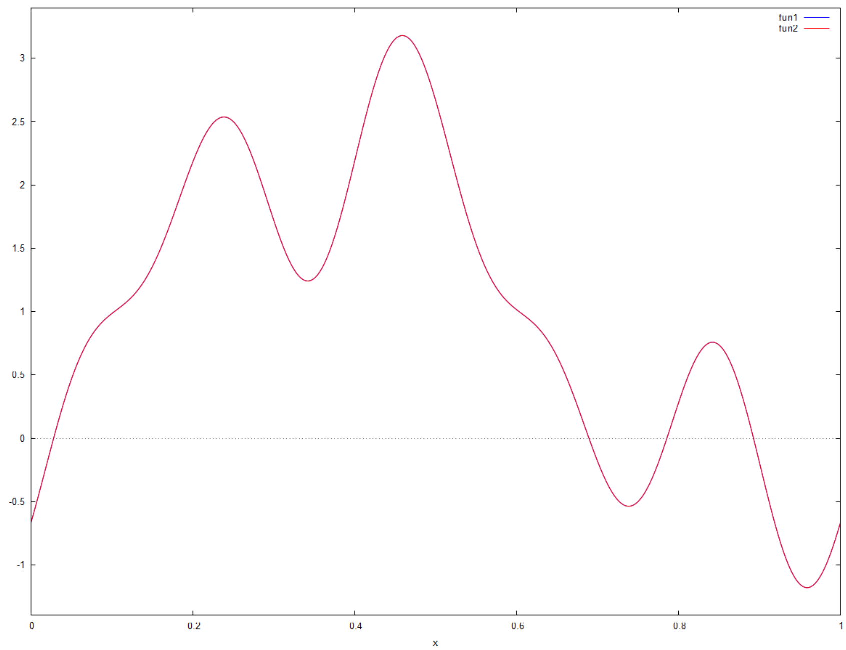
(%i157) f(x):=sin(2*%pi*x)-cos(2*%pi*x)+1+1/2*sin(10*%pi*x)-2/3*cos(6*%pi*x);

$$\mathrm{f}(x):=\sin(2\pi x)-\cos(2\pi x)+1+\frac{1}{2}\sin(10\pi x)+\frac{-2}{3}\cos(6\pi x) \qquad (\%\ o157)$$

(% i158) f_hat(x):=F(f(x),5,1);

$$f\_hat(x) := F(f(x), 5, 1) \qquad (\%\ o158)$$

(% i159) wxplot2d([f(x),f_hat(x)],[x,0,1]);




(% t159)

(% o159)

(% i160) g(x):=(4*x*(1-x))^ 5;

$$g(x) := (4x\ (1-x))^5 \qquad (\%\ o160)$$

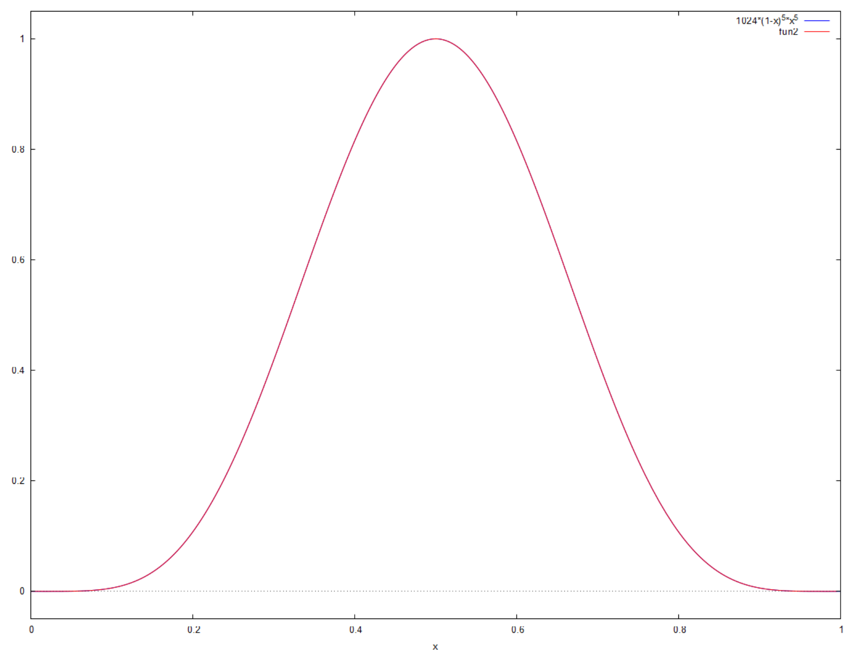
(% i161) g_hat(x):=Fn(g(x),10,1);

$$g\_hat(x) := Fn(g(x), 10, 1) \qquad (\%\ o161)$$

**(% i162)** g_hat(x);

$$- \left(7.9181463611022610^{-16} \sin\left(20\pi x\right)\right) - 7.14632186445823810^{-6} \cos\left(20\pi x\right) + 8.12577507237865210^{-16} \sin\left(18\pi x\right)$$ (% o162)

**(% i163)** wxplot2d([g(x),g_hat(x)],[x,0,1]);




(% t163)

(% o163)

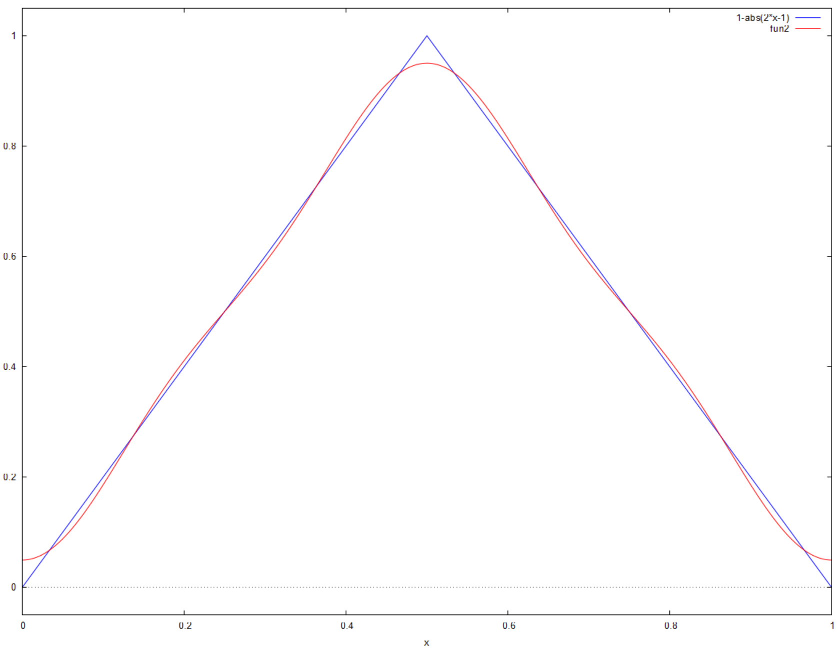
**(% i164)** h(x):=1-abs(2*x-1);

$$\mathrm{h}(x) := 1 - |2x - 1|$$ (% o164)

**(% i172)** h_hat(x):=Fn(h(x),3,1);

h_hat$(x) :=$ Fn (h$(x)$ , 3 , 1) (% o172)

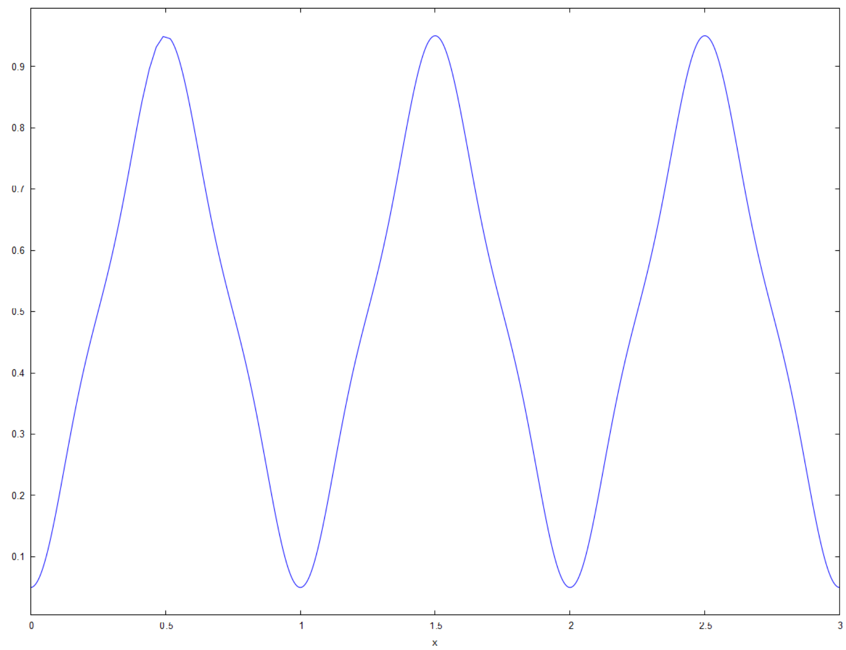
(% i173) wxplot2d([h(x),h_hat(x)],[x,0,1]);




(% t173)

(% o173)

(% i174) wxplot2d(h_hat(x),[x,0,3]);



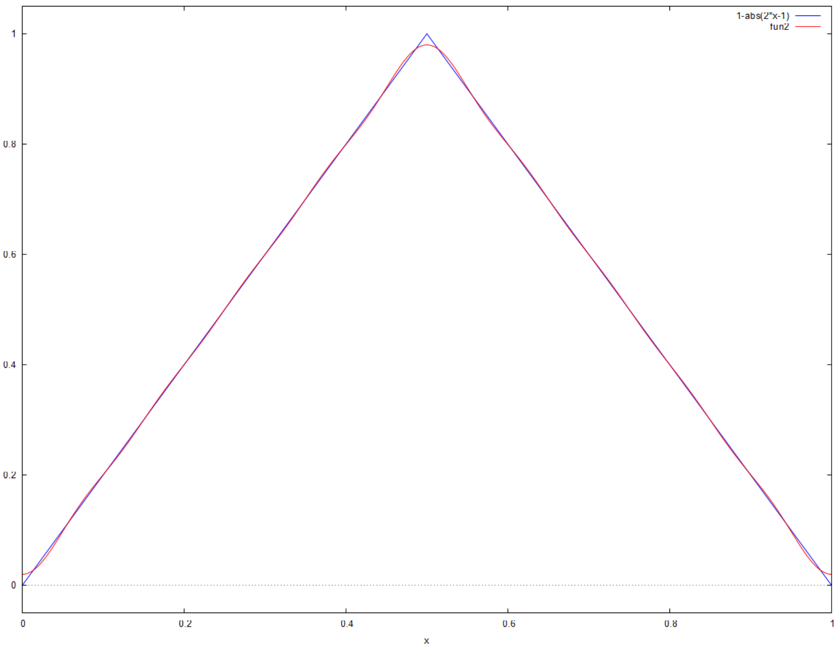

(% t174)

(% o174)

(% i175) h_hat(x):=Fn(h(x),10,1);

$$h\_hat(x) := Fn\left(h(x), 10, 1\right) \quad (\% \ o175)$$

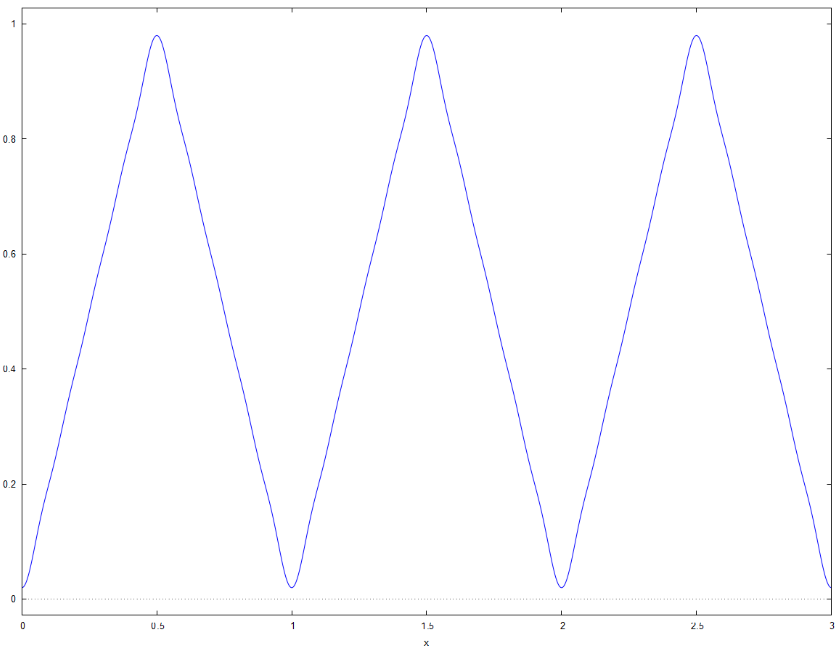
(%i176) wxplot2d([h(x),h_hat(x)],[x,0,1]);




(%t176)

(%o176)

(%i177) wxplot2d(h_hat(x),[x,0,3]);




(%t177)

(%o177)